

**PERTURBATION METHOD IN THE DIFFRACTION PROBLEM OF ALMOST
GRAZING INCIDENT PLANE WAVE BY AN ANISOTROPIC IMPEDANCE
WEDGE**

Anizotropik Bir Kama Yüzeyine Eğik Bir Şekilde Gelen Düzlem Bir Dalganın
Kırınımı Problemi İçin Perturbasyon Metodu

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ABSTRACT

In the present work, the problem of plane wave diffraction by a wedge with an anisotropic impedances is investigated for the case of almost grazing incidence. Wedge is a canonic structure and diffraction from wedge may be used in modelling scattering from a variety of complex structures. In this study, by using the Maxwell's equations the field components can be expressed in terms of z-components. By applying appropriate boundary conditions, a coupled system of equations is obtained in terms of field component and derivatives of field components with respect to φ and r. By using similarity transform to the coupled system of equations, the coupling is reduced to the simplest form in which Malyuzhinets theorem can be applied. The solution of field components is sought in the form of Sommerfeld integrals. The Malyuzhinets theorem is applied to the Sommerfeld integrals. By using Sommerfeld integrals the problem is reduced to a system of coupled functional equations. Solution of homogeneous functional equations is given in terms of χ_ϕ functions. For a small parameter of the problem ($\sin\theta_0 \ll 1$ where θ_0 is the angle between z-axis and incident wave) the perturbation procedure is used to reduce the coupled functional equations to a system of linear equations with this small parameter being at the integral terms of equations. As a result the closed form solution is given for functional equations. The obtained analytic expression for the spectral functions is substituted to the Sommerfeld integrals, which are evaluated by means of steepest descent technique. Then, the analytical expressions for the diffraction coefficient for both magnetic and electric field components are derived.

Key Words: Impedance Wedge, Functional Equations, Sommerfeld Integrals, Maliuzhinets Theorem, Perturbation Procedure

ÖZET

Bu çalışmada ayrıta çok küçük bir açıyla gelme durumunda, düzlemsel dalgaların kamadan kırınımı problemi incelenmiştir. Kanonik bir yapı olan kamadan

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kırınım çeşitli karmaşık nesnelerin saçılma modellemelerinde kullanabilir. Maxwell denklemleri ile, alan bileşenleri z-bileşenleri cinsinden ifade edilmiştir. Uygun sınır koşulları kullanılarak, alan bileşenleri ve onların ϕ ve r 'ye göre türevleri cinsinden bir kuple diferansiyel denklem sistemi elde edilmiştir. Benzerlik dönüşümü kullanılarak, kuple denklem sistemi, Maliuzhinets teoreminin uygulanabileceği en basit forma indirgenmiştir. Alan bileşenleri için çözüm Sommerfeld integralleri ile ifade edilip kuple denklemlerde yerine koyularak elde edilen integral eşitlik sistemine Maliuzhinets teoremi uygulanmış ve denklem sistemi bir kuple fonksiyonel denklem sistemine indirgenmiştir. Problemin küçük bir parametresi için ($\sin\theta_0 \ll 1$: θ_0 :gelen dalga ile z-ekseni arasındaki açı) perturbasyon metodu kullanılarak kuple fonksiyonel denklem sisteminin bir yaklaşık çözümü elde edilmiştir. Fonksiyonel denklem sisteminin homojen çözümü χ_ϕ fonksiyonları cinsinden ifade edilmiştir. Spektral fonksiyonlar için elde edilen analitik ifadeler en dik iniş çizgisi yöntemiyle çizilen Sommerfeld integrallerinde kullanılmış ve böylece hem manyetik alan hem de elektrik alan bileşenlerine ilişkin difraksiyon katsayılarının analitik ifadeleri elde edilmiştir.

Anahtar Kelimeler: Empedans Kama, Fonksiyonel Eşitlikler, Sommerfeld İntegralleri, Maliuzhinets Teoremi, Perturbasyon Metodu

Introduction

When an electromagnetic wave strikes an object it is scattered by this object. The scattered wave can be used to determine some properties of the scatterer such as the physical parameters, the shape, the velocity ect. of the obstacle and it is widely used in military and civil application such as radar and medical applications.

The problem of the determination of the scattering waves from complex shaped objects are not easy for each case. But for some well-defined geometrical structures such as sphere, cylinder, strip, half-plane, wedge, named as canonical structures the scattered waves would be obtained at least on some certain conditions. The solutions for such canonical structure are important, because the results may be used for the solution of scattering from complex structures.

In many practical applications, scatterers are at least partly wedge shaped metallic structures covered by dielectric materials or metallic structures with finite conductivity which can be simulated with impedance boundary conditions. Scattering by an impedance wedge was solved only for some simplified case, such as normal incidence, some specific values of wedge opening angle, some specific values of surface impedance of the wedge surface.

In this study the perturbation method will be used to determine the diffraction coefficients for an anisotropic impedance wedge when the incident wave almost graze the edge of the wedge.

For this purpose the computer codes will be written and the results will be represented graphically.

Material and Method

The problem under consideration is a wedge with a wedge opening angle 2Φ , where the edge coincides with the z-axis. The direction of propagation of incidence wave is specified by the angles θ_0 and ϕ_0 as shown in figure1.

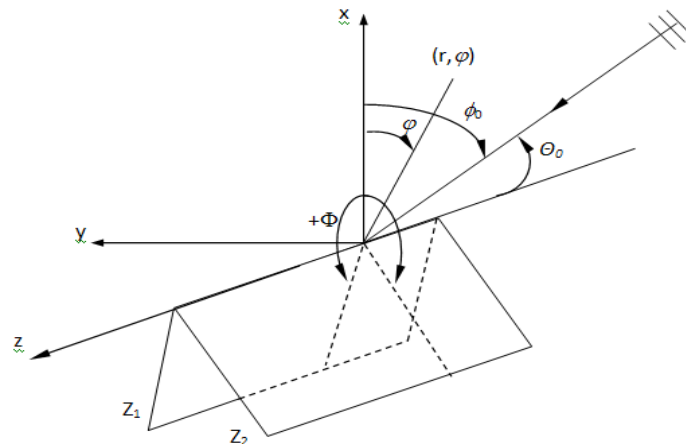


Figure 1. The Geometry of the Problem

The problem of scattering of plane waves by an impedance wedge was solved by Maliuzhinets for normal incidence case (Maliuzhinets,1950-1955).In this method the total field is expressed in terms of an integral with an unknown spectral function. By applying the boundary conditions, an integral equation is obtained and is transformed into a first order functional equation. After the determination of the unknown spectral function by solving the functional equation, the integral representation of the field is evaluated asymptotically.

In the case of oblique incidence, scattering problem by a wedge was not solved explicitly due to the resultant system of functional equations for two unknown functions. These equations can not be decoupled in general case. However, for some specific values of the wedge surface impedance and for some special values of the wedge opening angle, the analytic solution have been obtained (Williams 1959 and Senior,1986).

In this study, the case of almost grazing incidence will be investigated. By using Sommerfeld integrals the problem will be reduced to a system of coupled functional

equations. For a small parameter of the problem ($\sin\theta_0 \ll 1$ where θ_0 is the angle between z-axis and incident wave shown in fig.1) the perturbation procedure will be used to reduce the coupled functional equations to a system of linear equations with this small parameter being at the integral terms of equations. The obtained analytic expression for the spectral functions will be substituted to the Sommerfeld integrals, which are evaluated by means of steepest descent technique. The result related by the diffraction coefficient, expressed in terms of spectral functions will be represented.

For this purpose the computer codes will be written and the results will be represented graphically.

The Aim of The Study

The structure under the study is widely used for practical structure, but scattering by wedge was solved only for limited cases, such as certain opening angles of the wedge, certain incidence angles and some specific values of wedge surface impedance. The aim of this study is enlarge the class solvable cases, i.e, for an arbitrary wedge opening angle and arbitrary anisotropic surface impedance having axial-symmetry, but almost grazing incidence case.

Although this study does not solve the general case, the solution found here should be at least useful for comparison purposes for more general analytic solutions to be found and for checking the other numerical methods proposed for investigating wave diffraction by anisotropic impedance wedges.

Numerical Results

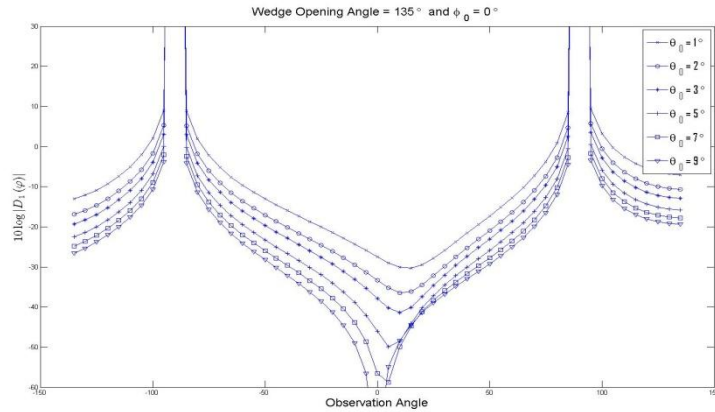


Figure 2: Diffraction coefficient $10\log_{10} |D_1(\varphi)|$ versus observation angle with $\Phi=135^\circ, \varnothing_0=0^\circ, \theta_0=1^\circ, 2^\circ, 3^\circ, 5^\circ, 7^\circ, 9^\circ$
 $a_{12}^+ = 1.0, a_{12}^- = 1.0, a_{21}^+ = 1.0, a_{21}^- = 1.0$

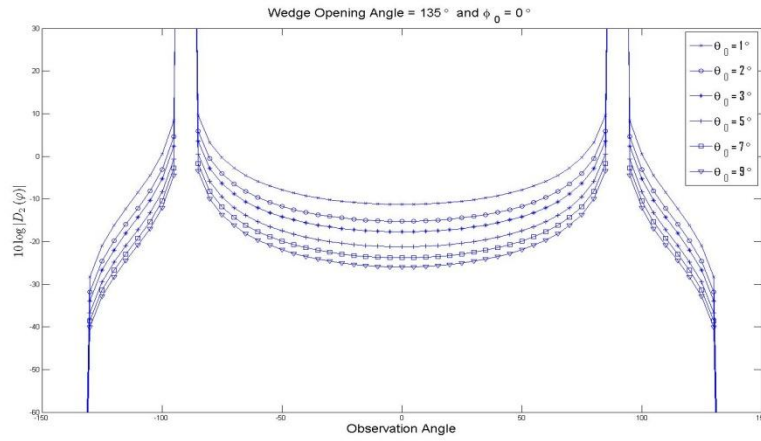


Figure 3: Diffraction coefficient $10\log_{10} |D_2(\varphi)|$ versus observation angle with $\Phi=135^\circ, \varnothing_0=0^\circ, \theta_0=1^\circ, 2^\circ, 3^\circ, 5^\circ, 7^\circ, 9^\circ, a_{12}^+ = 1.0, a_{12}^- = 1.0, a_{21}^+ = 1.0, a_{21}^- = 1.0$

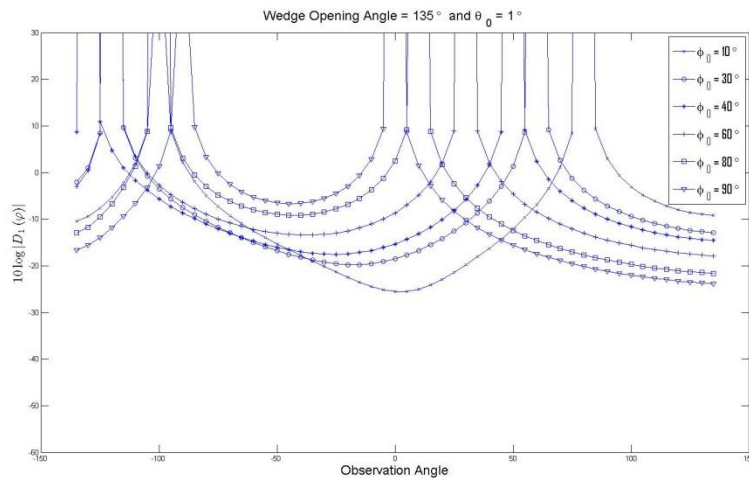


Figure 4: Diffraction coefficient $10\log_{10} |D_1(\varphi)|$ versus observation angle with $\Phi=135^\circ$, $\theta_0=1^\circ$, $\varnothing_0=10^\circ, 30^\circ, 40^\circ, 60^\circ, 80^\circ, 90^\circ$
 $a_{12}^+ = 1.0, a_{12}^- = 1.0, a_{21}^+ = 1.0, a_{21}^- = 1.0$

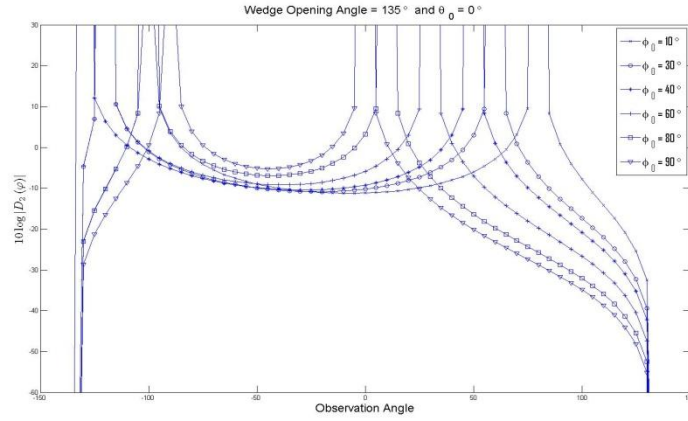
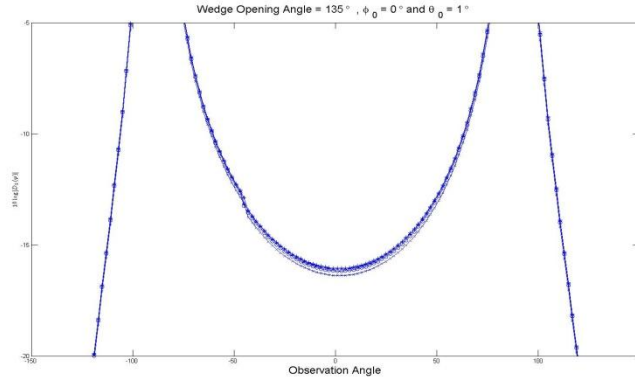


Figure 5: Diffraction coefficient $10\log_{10} |D_2(\varphi)|$ versus observation angle with $\Phi=135^\circ$, $\theta_0=0^\circ$, $\varnothing_0=10^\circ, 30^\circ, 40^\circ, 60^\circ, 80^\circ, 90^\circ$
 $a_{12}^+ = 1.0, a_{12}^- = 1.0, a_{21}^+ = 1.0, a_{21}^- = 1.0$



x	o	$*$	$+$
$a_{12}^+ = 0.5 + 0.2i$	$a_{12}^+ = 0.5 + 0.2i$	$a_{12}^+ = 5.0 + 0.2i$	$a_{12}^+ = 0.5 + 0.2i$
$a_{12}^- = 2.4 + 3.2i$	$a_{12}^- = 2.4 + 3.2i$	$a_{12}^- = 2.4 + 3.2i$	$a_{12}^- = 2.4 + 3.2i$
$a_{21}^+ = 2.0 + 4.0i$	$a_{21}^+ = 4.0 + 6.0i$	$a_{21}^+ = 6.0 + 8.0i$	$a_{21}^+ = 8.0 + 10.0i$
$a_{21}^- = 2.0 + 4.0i$	$a_{21}^- = 4.0 + 6.0i$	$a_{21}^- = 6.0 + 8.0i$	$a_{21}^- = 8.0 + 10.0i$

Figure 6: Diffraction coefficient $10\log_{10} |D_2(\varphi)|$ versus observation angle with $\Phi=135^\circ, \theta_0=0^\circ, \theta_0=1^\circ$

Conclusion

In the present study, the diffraction of plane waves by an anisotropic impedance wedge for almost grazing incidence case is considered. Even though there are numerous studies about the effects of impedance wedge on the propagation of electromagnetic waves, for some specific wedge opening angle, surface impedance and incidence angle, the diffraction from an arbitrary wedge for almost grazing incidence case is being investigated for the first time in this study. For a small parameter of the problem ($\sin \theta_0 \ll 1$, where θ_0 is the angle between z-axis and incident wave), the perturbation procedure enable us to reduce the coupled functional equations to a set of linear equations.

From equation (1), it is obvious that the diffracted magnetic and electric fields are the functions of the derived diffraction coefficients D_1 and D_2 . Rewriting this equation in the form of;

$$(Z_0 H_z^d, E_z^d) = \frac{e^{ik_0 r}}{\sqrt{r}} P_{diag} \begin{pmatrix} D_1(\varphi) \\ D_2(\varphi) \end{pmatrix} P^{-1} \bar{V}_0 \quad (1)$$

We can get the diffraction coefficients for magnetic field and electric field separately as follows,

$$D_H = D_{HH} + D_{HE} = -(D_1 + D_2)u_{10} + i(D_2 - D_1)u_{20} \quad (2)$$

and

$$D_E = D_{EE} + D_{EH} = -(D_1 + D_2)u_{20} + i(D_1 - D_2)u_{10}. \quad (3)$$

Here D_{HH} and D_{EE} denote the co-polarized components while the terms D_{HE} and D_{EH} corresponds to the cross-polarization components. Expressing the diffraction coefficient in this form enable one to see the cross and co-polarization effect of the wedge on the diffracted field. For example for the TE case ($E_z=0$, $u_{20}=0$) $D_H=D_{HH}=-(D_1+D_2)$ and $D_E=D_{EH}=i(D_1-D_2)$ and similarly, for TM case ($H_z=0$, $u_{10}=0$) $D_H=D_{HE}=i(D_2-D_1)$ and $D_E=D_{EE}=-(D_1+D_2)$. But these two special cases correspond to the normal incidence case, in which the functional equations can be decoupled and the exact solution may be derived easily. Hence the cross and co-polarized component is not considered separately in this study.

In figure 2 the variation of diffraction coefficient D_1 with respect to the observation angle for different opening angles and skewness angles are given. As shown in this figures, D_1 decrease dramatically with increasing skewness angle, while there is no such a dependence between D_2 and skewness angle as shown in figure 3.

In figures 4 and 5 the variation of D_1 and D_2 with respect to ϕ_o are given.

While ϕ_o increases, both of D_1 and D_2 also increase.

In figure 6 the variations of D_1 and D_2 with respect to the surface impedances are given. While values of surface impedances increase both of D_1 and D_2 increase also.

It is obvious that for the case of $\Phi=90^\circ$ the geometry is reduced to an impedances junction while for $\Phi=180^\circ$, it is reduced to an half plane with anisotropic surfaces impedances. Using these values of opening angles, the results for the related geometries can be obtained easily.

Considering these different geometries and small skewness angle, we conclude that this approach enlarge the class of solvable diffraction problem in a small range. Additionally, we hope that, the results are valuable for the comparison purposes for the other approximate methods.

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