MESH SENSITIVITY STUDIES FOR MIXED-MODE TWO DIMENSIONAL FRACTURE PROBLEMS USING DISPLACEMENT CORRELATION TECHNIQUE*

İki Boyutlu Karışık Modlu Kırılma Problemlerinin Deplasman Korelasyon Tekniği Kullanarak Ağ Duyarlılık Çalışmaları

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ABSTRACT

The displacement correlation technique is still one of the most commonly used numerical methods to compute stress intensity factors using the finite element method. Depending on settings and values of parameters, some variation on the computed stress intensity factor can be yielded. Therefore, in this study, sensitivity studies were performed for two-dimensional cracks under mode-I and mixed-mode loadings that cover most of two-dimensional cracks encountered in practical applications. These problems are; mode-I central crack and mixed-mode inclined central crack. As the numerical tool, the finite element software ANSYS™ was used and its KCALC™ command was employed for calculation of stress intensity factors. The parameters considered for near-crack-tip mesh sensitivity studies are, numbers of elements in the circumferential direction, numbers of elements in the radial direction, sizes of elements along the crack faces, element size ratio in the radial direction and the size of first radial element in the crack zone. Conclusions were drawn by comparing the values obtained from two different mesh models with results from the literature and mesh parameter settings were recommended for accurate computation of stress intensity factors.

Key Words : Mixed-Mode Cracks, Displacement Correlation Technique, Finite Element Method, Fracture Mechanics

ÖZET

Deplasman korelasyon tekniği sonlu elemanlar metodunu kullanarak gerilme şiddet fatörlerinin hesaplanması için hala yaygın olarak kullanılan nümerik metodlardan biridir. Parametre değerlerinin ayarlanmasına ve değerlerine bağlı olarak gerilme şiddet faktörünün hesaplanmasında bazı varyasyonlar elde edilebilir. Bu nedenle bu çalışmada, duyarlılık çalışmaları pratik uygulamalarda karşılaşılan iki boyutlu çatlakların çoğunu kapsayan mod-I ve karışık modlu yükler altında iki boyutlu çatlaklar için uygulanmıştır. Bu problemler, mod-I merkezi çatlak ve karışık modlu eğik merkezi çatlak. Sayısal araç olarak, sonlu elemanlar yazılımlı ANSYS™ kullanılmıştır ve gerilme şiddet faktörlerinin hesaplanması için bu yazılımın KCALC™ komutu kullanılmıştır. Çatlak çevresinde ağ duyarlılık

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* MSc Thesis-Yüksek Lisans Tezi

çalışması için kabul edilen parametreler, dairesel yöndeki eleman sayıları, radyal yöndeki eleman sayıları, çatlak yüzeyi boyunca elemanların boyutu, radyal yöndeki eleman boyut oranı ve çatlak bölgesindeki ilk radyal eleman boyutudur. Sonuçlar gerilme şiddet faktörünün doğru hesaplanması için tavsiye edilen ağ parametreleri ve literatürdeki sonuçlar ile iki farklı ağ modelinden elde edilen değerlerin kıyaslanmasıyla belirlenmiştir.
Anahtar Kelimeler : Karışık Modlu Çatlaklar, Deplasman Korelasyon Tekniği, Sonlu Elemanlar Yöntemi, Kırılma Mekaniği

Introduction

Studying fracture of materials and structures is essential in order to predict life and safety. Cracks in structures initiate in different ways, such as surface or corner cracks, or embedded cracks. Fracture mechanics is important for most special areas such as industries of aircraft, spaceship, ship, automotive, etc. Therefore, fracture mechanics is used to check the integrity of a structure containing a crack and its damage tolerance.

There are some important and costly fracture incidents from the history, These failures could not be explained by traditional design or failure criteria. After Inglis' (1913), work on the stress concentrations for circular and elliptical holes, Griffith (1921, 1924), proved the size effect on the failure stress of a structure with crack. Detailed analytical methods followed Griffith's work that showed in detail the stress, strain and displacement fields near a two dimensional crack tip. See for example, Westergaard (1931, 1935), and Williams (1953). Of course, these analytical solutions could only be applied to problems with simple geometry and boundary conditions. Later, numerical methods had also started to be developed. In the mid 1970s, Barsoum (1974), Hensell and Shaw (1975), independently discovered that by taking the midside nodes of an element that are adjacent to a crack tip and moving them to quarter point of element side, the singular stress field which occurs at a crack tip could be simulated using a quadratic finite element. To determine stress intensity factors, the Displacement Correlation Method with a linear extrapolation from two nodes at each crack face was used by Mi (1996), and Shih et al. (1977). Fehl (1999), employed displacement correlation technique and the quarter-point displacement technique to solve fracture problems. He evaluated two nodal displacement methods used for prediction of cracking in massive concrete structures to predict K_I *and* K_{II}, respectively. Vorel (2001), employed numerical programs like Franc2d, Franc3d and MARC in his studies and determined stress intensity factors for modeling various fracture phenomena, such as fatigue crack growth by displacement correlation method. Later, Park (2004), developed a finite element modeling approach for the mixed mode 2D linear elastic fracture mechanics crack propagation analysis based on the displacement correlation method. Souiyah (2007), presented the calculation and comparison of the SIFs for a cracked plate by using several different numerical techniques such as Element Free Galerkin Method (EFGM).

The displacement correlation technique is still one of the most commonly used methods to compute stress intensity factors using the finite element method. Depending on settings and values of parameters, the method can yield some variation on the computed stress intensity factor. Therefore, in this study, sensitivity studies are performed for different two-dimensional cracks under mode-I and mixed-mode loadings that cover most of two-dimensional cracks encountered in practical applications. These problems are; mode-I and mixed-mode central cracks under uniform stress loading, mode-I edge cracks under uniform tensile stress and bending loads. As the numerical tool, the finite element software ANSYS™ is used and its KCALC™ command is employed. The above problems are studied by developing macros that generate the models, solve them and perform post-processing of results automatically. The computed stress intensity factors from two types of mesh models for different settings of parameters are compared to known analytical solutions. These two models are "KSCON Model" and "Circular Zone Model". The corresponding sensitivity graphs are made to visually see the parameters that affect results the most. Conclusions are drawn between the two mesh models and the recommended parameter settings are explained.

Material and Method

Analysis of Fracture in Plates

Fracture problems are encountered in many technology fields such as space industry, aviation industry, shipping industry, defense industry, etc. because use of high technology in these fields is required. Materials used in these areas are exposed to many factors such as high temperature, strong impact, high corrosion, etc.

Fracture can occur inside the material or at the edges of a part. Plates are also a wide class of materials used in engineering applications. Therefore, in this study, the focus is give to various two-dimensional crack problems in plates. As the numerical tool for modeling, the finite element software ANSYS™ is used.

ANSYS™

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ANSYS™ Theory Manual provides the theoretical basis for calculations performed within ANSYS™ program, such as elements, solvers and results formulations, material models, and analysis methods. By understanding the underlying theory, we can make better use of ANSYS™ capabilities while being aware of assumptions and limitations. The manuals listed below form the ANSYS™ product documentation set. They include descriptions of the procedures, commands, elements, and theoretical details needed to use ANSYS™.

Crack Tip Mesh Models Studied

Accuracy of results are very important in numerical studies. Therefore, in this study, two different models, which are called Circular Zone Model" and "KSCON™ Model", are developed and macros are written that develop models, solve them and post-process them automatically to calculate the mixed-mode stress intensity factors.

Circular Zone Model

This model was created by using ANSYS™ commands. First, parameters were listed that possibly affect the stress near crack tip. Later, automated macros were created that automatically controls the mesh near crack tip.

Crack zone of this model was created from two parts to obtain regular meshes and precise results (Figure 1). The first part of crack zone was created from three circles by KSCON command in ANSYS™ . It was generated as mapped mesh by MSHKEY command in ANSYS™. The number of elements in radial direction is fixed for this region. In the second circular zone the number of elements in radial direction can be changed. It was generated as free mesh by Amesh command in ANSYS™. In this way, the mesh is generated as more controlled. Other parameters such as global element size were made independent in order not to affect results from meshes of other regions.

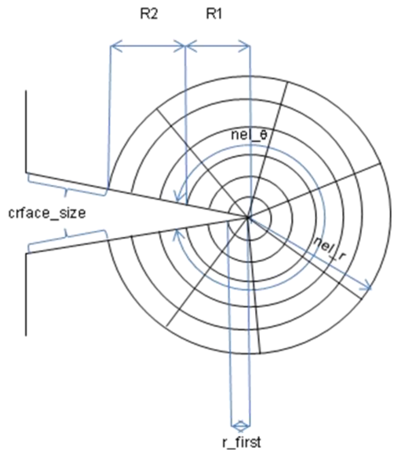


Figure 1. The full view of Circular Zone Model

Parameters selected are nel_θ, nel_r, crface_size, R_Size_Ratio= R2/R1, r_first. nel_θ is the number of elements in θ (circumferential) direction. nel_r is the number of elements in radial direction. crface_size is elements size on the crack surfaces (Figure 1). R_Size_Ratio is ratio of R2/R1. It represents the ratio of radial sizes of the two zones. r_first represents radial edge length of triangular elements in the first row that touch the crack tip.

KSCON™ Model

This model was also created using ANSYS™ commands. Crack zone of this model was created as only one part (Figure 2). Because KSCON™ model is simpler than Circular Zone Model. Crack zone was created as one part and the mesh was generated as free mesh by Amesh command of ANSYS™. The number of circular elements in radial direction is two and therefore, parameter nel_r does not exist in this model. Other parameters such as global element size was also included in this model.

Parameters selected are nel_θ, crface_size, R_Size_Ratio= R2/R1 and r_first. nel_θ is the number of elements in θ (circumferential) direction. crface_size is element size on crack surfaces. R_Size_Ratio is R2/R1. It represents the ratio of edge sizes of second and first element in radial direction near the crack tip. r_first represents edge length first row triangular elements in radial direction that touch the crack tip.

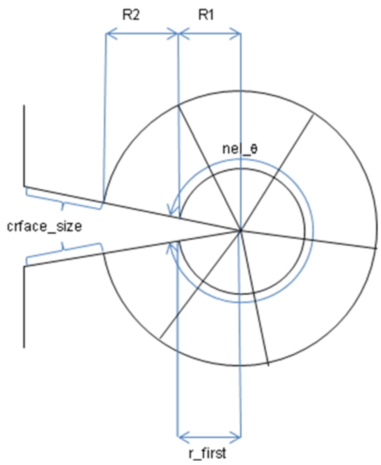


Figure 2. The KSCON model

Results and Discussion

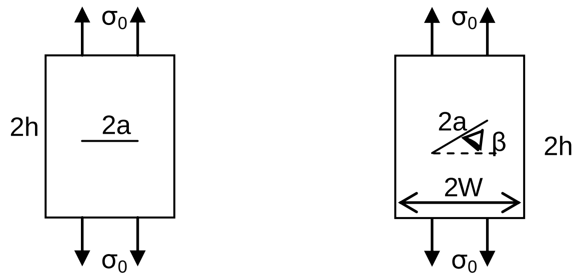
Two Dimensional Problems Studied

Intro-Brief Description of Problems

Cracks in plates can occur in different shapes because of particular reasons. Depending on their shape and loading conditions, they are examined in two or three dimensions. Many crack problems in plates can adequately be modeled in two dimensions (2-D). In this thesis, the following 2-D fracture problems are studied:

1. Mode-I central crack under uniform tension loading (σ_0) (Figure 3a)
2. Mixed-mode inclined central crack under uniform tension loading (Figure 3b)

Specifically, crack region mesh sensitivity studies are performed. In these figures, 2h is the plate height, 2W is its width, 2a is crack length, β is crack inclination angle and σ_0 is the applied remote normal stress acting in y-direction. First, a plate size analysis was done for the mode-I central crack problem to make sure that the infinite plate condition is adequately satisfied. It is observed that the stress intensity factor results don't change after W=h=40a, which is the plate size taken for all cases. σ_0 was taken 1 Pa ;



(a) **(b)**

Figure 3. (a) Mode-I central crack under uniform tension loading, (b) Mixed-mode inclined central crack under uniform tension loading

Each of the problems shown was studied using a concentrated mesh and different finite element mesh refinement cases on stress intensity factors were investigated. Having compared these solutions with theoretical results, conclusions were drawn and optimum mesh parameter settings were determined to obtain accurate SIFs using the displacement correlation technique (KCALC command in ANSYS©).

Mode-I Central Crack Uniform Tension Loading

A crack of 1m was placed at the center of the plate. Half of the model was considered and symmetry boundary conditions were applied for displacements in *x*-direction. W=h=40m and E.S. (global element size)=0.6 were taken for this model. The theoretical stress intensity factor for Mode-I central crack was calculated by equation (1) (Irwin et al., 2000).The results as graph were shown in Figure 4a, 4b.

$$K_I = \sigma\sqrt{\pi a} \quad (1)$$

Two different macros, which were called "Circular Zone model" and "KSCON™ model", were written. Size parameters were fixed and other mesh parameters for the crack region was considered. These are: nel_θ, nel_r, crface_size, R_Size_Ratio, r_first for Circular Zone model and nel_θ, crface_size, R_Size_Ratio, r_first for KSCON[TM] model.

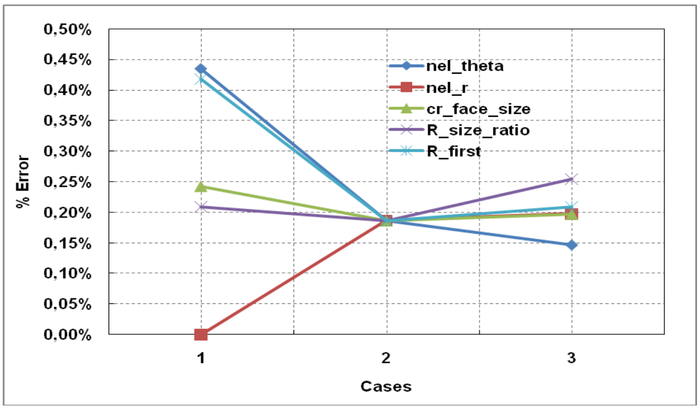


Figure 4a. Effect of meshing parameters, mode-I central crack under uniform tension loading, Circular Zone model, K1

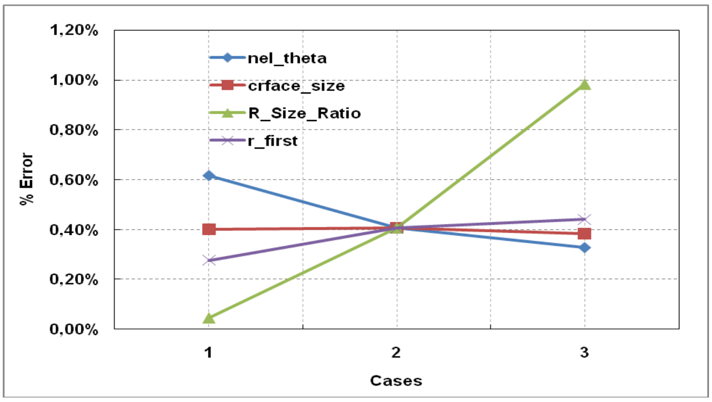


Figure 4b. Effect of meshing parameters, mode-I central crack under uniform tension loading, KSCON[TM] model, K1

When middle values of the parameters were used, K1 is within 0.19% of the theoretical value for the Circular Zone model and within 0.41% for the KSCON[TM] model. On the other hand, when the best parameter values were used, the results are within 0.0% for the Circular Zone model and within 0.05% for the KSCON[TM] model compared to the theoretical value . It is also seen that nel_θ and r_first are the parameters which influence the results the most for the Circular Zone model. It is also seen that nel_θ and R_Size_Ratio are the parameters which have the highest impact on K1 for the KSCON[TM] model.

Mixed-Mode Inclined Central Crack Under Uniform Tension Loading

Mixed-mode inclined central crack under uniform tension loading was the second problem studied. Since the model does not have any symmetry, the full model was considered. Cracks were placed at the center of the plate with certain angles and suitable boundary conditions were applied. Two different macros, which were called “Circular Zone model” and “KSCON^{TM} model”, were written. Also, β angles ranging from 0^0 to 90^0 with increments of 15^o were taken into account. Plate and element size parameters were fixed at W=h=40a and E.S.=0.6. Theoretical stress intensity factors which are given by equations (2) and (3) (Irwin et al., 2000).

$$K_I = \sigma\sqrt{\pi a}\,\sin\beta^2 \quad (2)$$

$$K_{II} = \sigma\sqrt{\pi a}\,\sin\beta\cos\beta \quad (3)$$

For 45^0 inclination angle case that, when parameter middle values are used for K1 in Circular Zone model, the results are within 0.34% of the theoretical value. For the case of best settings of parameters in the Circular Zone model, mode I stress intensity is within 0.27 % of the theoretical value. In the same way, when parameter middle values are used for K1 in KSCON^{TM} model, the results are within 0.28 % of theoretical value. For the case of best setting of parameters in the KSCON^{TM} model, mode I stress intensity is within 0.02 % of the theoretical value. nel_θ, nel_r and r_first are parameters which influences the results the most for Circular Zone model. For the KSCON^{TM} model, R_Size_Ratio, nel_θ and crface_size are parameters which influences the results the most. when parameter middle values are used for K2 in Circular Zone model, the results are within -0.07 % of the theoretical value. For the case of best settings of parameters in the Circular Zone model, mixed mode stress intensity is within 0.01 % of the theoretical value. In the same way, when parameter middle values are used for K2 in the KSCON^{TM} model, the results are within -0.44 % of the theoretical value. For the case of best settings of parameters in the KSCON^{TM} model, the results are within 0.09 % of the theoretical value. It is also seen that nel_θ and r_first are the parameters which influence the results the most for the Circular Zone model. R_Size_Ratio, nel_θ and crface_size are the parameters which influence the results the most for the KSCON^{TM} model.

Conclusion

When the results are reviewed, it can be seen that the computed stress intensity factors differ between Circular Zone model and KSCON^{TM} model. Percent absolute error average, as seen for both K1 and K2 in Table 2, is lower for Circular Zone model. In addition, the maximum error is also lower for both modes in Circular Zone model. Percent error distance (range) value of KSCON^{TM} model is bigger as seen in Table 1. Although average percent errors are lower for KSCON^{TM} model, this can be misleading. Because negative and positive percent errors can cancel each other. Therefore, absolute percent error values should be used to assess the quality and goodness of the two models. According to the above statistical comparisons, it is concluded that Circular Zone model yields better results. It is expected that the Circular Zone model can give better results because of higher degree of control and more quality mesh near crack tip. At the same time, it is also observed that if the middle parameter values are used, the results of Circular Zone model and KSCON^{TM} model are close to each other.

Table 1. % Error descriptive statistics table

	CZ_K1	CZ_K2	KSCON_K1	KSCON_K2
Average	0,56%	-0,09%	0,42%	-0,44%
Standard error	0,20%	0,04%	0,26%	0,02%
Median	0,33%	-0,07%	0,31%	-0,44%
Kip				
Standard deviation	0,81%	0,08%	0,98%	0,05%
Sample variance	0,01%	0,00%	0,01%	0,00%
Oblateness	151,26%	-257,30%	265,78%	-211,66%
Distortion	141,41%	-18,53%	49,79%	10,78%
Distance	2,93%	0,18%	4,35%	0,11%
Minimum	-0,40%	-0,18%	-1,63%	-0,49%
Maximum	2,53%	0,00%	2,72%	-0,38%
Total	8,92%	-0,46%	5,86%	-2,18%
Number of data	16	5	14	5

Table 2. % Absolute error descriptive statistics table

	CZ_K1	CZ_K2	KSCON_K1	KSCON_K2
Average	0,66%	0,09%	0,72%	0,44%
Standard error	0,18%	0,04%	0,21%	0,02%
Median	0,39%	0,07%	0,34%	0,44%
Kip	0,40%			
Standard deviation	0,73%	0,08%	0,77%	0,05%
Sample variance	0,01%	0,00%	0,01%	0,00%
Oblateness	228,56%	-257,30%	244,69%	-211,66%
Distortion	182,00%	-18,53%	173,19%	-10,78%
Distance	2,51%	0,18%	2,59%	0,11%
Minimum	2,00%	0,00%	0,13%	0,38%
Maximum	2,53%	0,18%	2,72%	0,49%
Total	10,50%	0,46%	10,02%	2,18%
Number of data	16	5	14	5

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Acknowledgements

The support of this research by TÜBİTAK (The Scientific and Technological Research Council of Turkey) is gratefully acknowledged.